University of Arkansas – Fort Smith
5210 Grand Avenue
P.O. Box 3649
Fort Smith, AR 72913
479-788-7000

General Syllabus

MATH 3104 Foundations of Mathematics

Credit Hours: 4   Lecture Hours: 4   Laboratory Hours: 0

Prerequisite: MATH 2443 Discrete Mathematics I

Prerequisite or corequisite: MATH 2854 Calculus II

Effective Catalog: 2020~2021

I.   **Course Information**

   A. **Catalog Description**
   Introduces fundamental concepts that underlie all branches of mathematics, preparing students for advanced theoretical courses in mathematics. The primary emphasis will be learning to write valid proofs of mathematical propositions. Topics include propositional calculus, sets, basic number theory, order and equivalence relations, cardinality, field axioms, limits, and complex numbers.

II.   **Student Learning Outcomes**

   A. **Subject Matter**
   Upon successful completion of this course, the student will be able to:

   1. Recognize quantifications in propositions, and properly construct direct and indirect proofs of existential and universally quantified propositions.
   2. Write valid proofs regarding relations between sets, including infinite families of sets.
   3. Write valid proofs regarding order and equivalence relations.
   4. Prove basic properties of functions, including propositions about injectivity, surjectivity, and composition.
   5. Prove propositions comparing cardinalities of sets.
   6. Use mathematical induction to prove propositions regarding the natural numbers, including propositions involving inequalities.
   7. Identify whether operations on sets possess basic algebraic properties of closure, associativity, existence of identity elements and inverses, and distributive properties, and write proofs based on these properties.
   8. Write valid proofs involving limits of functions, including sequences.

9. Perform basic operations on complex numbers and write proofs of basic arithmetic properties of this field.

B. University Learning Outcomes
This course enhances student abilities in the following areas:

Analytical Skills
Critical Thinking Skills: Students will explore mathematical relationships, form conjectures regarding general principles, and deductively prove these conjectures. Students will analyze arguments and identify invalid conclusions and logical fallacies.
Quantitative Reasoning: Students will develop a fundamental understanding of numbers and the nature of mathematical reasoning, and what constitutes proof of a quantitative relationship.

Communication Skills (written and oral)
Students will communicate mathematically, proving results in a logically valid and deductively precise manner.

Ethical Decision Making
Students will explore the nature of deductive systems, and learn to recognize the dependence of conclusions on fundamental assumptions.

III. Major Course Topics

A. Propositional Calculus
   1. Quantified statements
   2. Valid argument forms
B. Sets and set operations
   1. Infinite families of sets
C. Relations
   1. Equivalence relations and partitions
   2. Order relations
D. Functions
   1. Injectivity and surjectivity
   2. Images and pre-images of sets
   3. Composition
E. Cardinality
   1. Finite and infinite
   2. Countable and uncountable
F. The natural numbers
   1. Principle of mathematical induction
      a. Weak and strong forms of the principle of mathematical induction
   2. Well-ordering principle
G. Algebra
   1. Properties of operations on sets

       2.   Field axioms
H. Analysis
       1.   Limits
           a. Functions
           b. Sequences
I.   Complex Numbers
       1.   Arithmetic operations
       2.   Algebraic properties