University of Arkansas – Fort Smith 5210 Grand Avenue P.O. Box 3649 Fort Smith, AR 72913 479-788-7000

General Syllabus

MATH 4303 Real Analysis I

Credit Hours: 3 Lecture Hours: 3 Laboratory Hours: 0

Effective Catalog: 2020-2021

Prerequisite: MATH 3104 Foundations of Mathematics

I. Course Information

A. Catalog Description

An introduction to the theory of calculus of one and several variables, limits, continuity, sequences, differentiation, integration, measure theory, and infinite series. Includes a brief introduction to metric spaces and general topological spaces.

B. Additional Information

This course is designed to provide students with a deeper understanding of many of the concepts encountered in the calculus sequence and to introduce related topics in real analysis and topology. Therefore, this course is beneficial for future secondary mathematics teachers who may find themselves teaching calculus and essential for students planning to continue their study of mathematics.

II. Student Learning Outcomes

A. Subject Matter

Upon completion of this course, the student will be able to:

- 1. State definitions and theorems involving properties of real numbers and their subsets such as the Archimedean Property, the density of the rational numbers, the field axioms, the Completeness Axiom and compactness and prove elementary theorems involving these concepts.
- 2. State definitions involving sequences of real numbers including limits and convergence and prove theorems involving sequences and their limits.
- 3. State definitions involving limits and continuity of functions and prove theorems related to these topics.
- 4. Demonstrate an understanding of the concepts of differentiation and integration and how the Fundamental Theorem of Calculus relates them.
- 5. Perform convergence tests on infinite series.

- 6. Distinguish between pointwise and uniform convergence for sequences of functions.
- 7. Communicate mathematics by producing readable and correct mathematical arguments in the form of formal mathematical proofs of various theorems in analysis.

B. University Learning Outcomes

This course enhances student abilities in the following areas:

Analytical Skills

Critical Thinking Skills: Students will apply the concepts of real analysis to the solution of problems. Students will develop their ability to discover conjectures and prove propositions. These proofs will enhance the students' deductive reasoning abilities. In addition, students will be required to create many examples and counterexamples to demonstrate their understanding of various topics in the class.

Communication Skills (written and oral)

Students will communicate mathematical concepts and justifications. They will learn the importance of precise statements of definitions, postulates, and propositions. Student will write mathematically accurate and clear proofs of theorems.

III. Major Course Topics

- A. Real numbers
 - 1. Archimedean Property
 - 2. Field Axioms
 - 3. Density of the Rationales
 - 4. Topology of the Reals
 - 5. Compactness, the Heine-Borel theorem
 - 6. Cantor's set
- B. Sequences
 - 1. Limits and convergence
 - 2. Monotone Convergence theorem
 - 3. Cauchy sequences and the Cauchy Convergence Criterion
 - 4. Bolzano-Weirstrass theorem
- C. Limits and Continuity of functions
 - 1. Limits of functions
 - 2. Properties of continuous functions
 - 3. Uniform continuity
- D. Differentiation
 - 1. The Mean Value theorem
 - 2. Taylor's Theorem
- E. Integration
 - 1. The Riemann Integral

- 2. The Fundamental Theorem of Calculus
- F. Infinite series
- G. Sequences and series of functions
 1. Pointwise and uniform convergence
 H. Topological spaces and metric spaces