University of Arkansas – Fort Smith
5210 Grand Avenue
P.O. Box 3649
Fort Smith, AR 72913
479-788-7000

General Syllabus

MATH 4853 Introduction to Topology

Credit Hours: 3 Lecture Hours: 3 Laboratory Hours: 0

Prerequisite: MATH 4303 Real Analysis I

Effective Catalog: 2018~2019

I. Course Information

A. Catalog Description

An introductory study of both point-set and algebraic topology. Point-set concepts covered include open, closed, finite, countable, and uncountable sets, order, product, subspace, metric, and quotient topologies, continuous functions, open and closed maps, homeomorphisms, connectedness, and compactness. Algebraic concepts include homotopy, homomorphism, chain maps, fundamental groups, homotopy groups, covering spaces, and homology.

B. Additional Information - None

II. Student Learning Objectives

A. Subject Matter

Upon completion of this course, the student will be able to:

1. State definitions and identify examples of open, closed, finite, countable, and uncountable sets.
2. Prove theorems about open, closed, finite, countable, and uncountable sets.
3. Construct product, subspace, metric, and quotient topologies on different sets and prove theorems about these topological spaces.
4. Build new topological spaces (and continuous functions) from ones already known.
5. Determine which topologies can be defined on a collection of subsets to form a topological space.
6. Prove basic theorems about general topological spaces.
7. Give examples of continuous functions, open and closed maps, and homeomorphisms

8. Determine whether a given function is continuous, open, closed, or a homeomorphism, and prove theorems regarding these maps.
9. Use notions of compactness and connectedness to distinguish topological spaces up to homeomorphism
10. Give examples of topological spaces and prove theorems based on assumptions of connectedness and/or compactness.
11. Identify homotopic paths and construct homotopy mappings.
12. Use the notion of homotopic paths to define the fundamental group of a space.
13. Identify groups associated with topological spaces, and relate the algebraic properties of the group to topological properties of the space.
14. Classify topological spaces up to homeomorphism by their fundamental groups.
15. Construct a covering space for a given topological space and prove theorems related to the covering structure.
16. Construct homology modules and relate algebraic properties of homology modules to topological properties of spaces.

B. University Learning Outcomes

Analytical Skills
Critical Thinking Skills: Students will analyze various mathematical concepts that arise in the study of topology, draw conclusions, and make generalizations based on these concepts.

Communication (written and oral)
Students will communicate proficiently by composing coherent documents which are appropriate for the intended audience and will effectively communicate orally in a public setting, such as presentation for peers or at conferences.

III. Major Course Topics

A. Introductory set theory
   1. Finite and Infinite sets
   2. Countable and uncountable sets
   3. Open and closed sets

B. Topological spaces
   1. Continuous functions
   2. Homeomorphisms, open and closed maps
   3. Order topology
   4. Box and product topology
   5. Subspace topology
   6. Quotient topology
   7. Connectedness
   8. Compactness

C. Algebraic topology
   1. Homotopy
   2. Homotopy groups

3. Simplicial, cellular and singular homology
4. Suspensions, loop spaces and Fibre bundles